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TÍTULO: Representaciones de contenidos, aplicaciones prácticas y relaciones interdisciplinarias como núcleo de coherencia en Educación Matemática.

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RESUMEN: Este artículo aborda conexiones significativas entre representaciones de contenidos matemáticos, sus aplicaciones prácticas y relaciones interdisciplinarias. El estudio empírico se realizó durante cinco cursos académicos, en instituciones acreditadas del Sistema Nacional de Educación de Cuba. Los miembros de la muestra fueron 121 profesores de Matemática titulados de la provincia de Las Tunas. Se utilizó una escala estadística y un software especializado para evaluar adecuadamente la coherencia en la enseñanza de la Matemática. Se diseñó un modelo adecuado en dos subsistemas (organización y planificación). Se favoreció el proceso establecido al trabajar con sistemas de representación interdisciplinarios a partir de relaciones contextuales.

PALABRAS CLAVES: matemática, coherencia, interdisciplinariedad, representaciones, aplicaciones.

TITLE: Content representations, practical applications and interdisciplinary connections as mathematics teaching coherence core

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ABSTRACT: This article deals with meaningful connections among mathematical contents representations, their practical applications and interdisciplinary relations. The empirical study was carried out during five academic years, in accredited institutions of the Cuban National Education System. Sample members were 121 qualified mathematics teachers from Las Tunas province. A statistical scale and specialized software to properly evaluate mathematics teaching coherence were used. Results revealed that problems in these three key indicators represent the principal causes of inadequacies. A proper model was designed in two subsystems (organizing and planning). The established process was favored by working with interdisciplinary representation systems from contextual relationships.

KEY WORDS: mathematics, coherence, interdisciplinarity, representations, applications.

INTRODUCTION.

The effective teaching of mathematics is relevant and an essential priority, because it allows the potential acquisition of valuable content to provide satisfactory solutions to current complexities. In this regard, actual students should be holistic, critical and integrate into the social environment. Today's teachers must demand the necessary appropriation, broadening, deepening and practical application of curricular contents with a scientific conception. However, they merely present fragmented knowledge. It highlights the fundamental need to consider the complementary relationships with other disciplines to address them with comprehensive approaches.

Understanding the particular relationship of mathematics to other disciplines, makes it special for interdisciplinary work and education. This key idea should be based on solving problems, conscious reflection, exhaustive analysis of content meanings and representation forms. However, interdisciplinary mathematics teaching-learning process is established from content standards (Alsina, 2017). It is addressed with cognitive nodes (Monsalve and Carvajal, 2020), although it is typically limited to the logical framework of specific knowledge. The teaching solutions for a more comprehensive, coherent and organized work, modeling situations from various academic disciplines and contexts are insufficient. It would foster a need-to-understand and problem-solving attitude. Consequently, the genuine need to work with interdisciplinary approaches from mathematical process standards is evident.

Content simplification to make it more accessible to the students typically implies impoverishment of teaching interactions, and consequently, the considerable loss of knowledge essential characteristics (Barallobres, 2016).

It is necessary to identify the essential concepts properties through problem-solving processes. The gradual establishment of appropriate links among diverse representations should be involved. “Contemporary conceptualization of mathematical competencies and mathematical thinking underline the importance of knowledge of multiple representations” (Milinkovi´c, 2019, p.80). However, it is typical to find it isolated in a particular subject, like the variety of representations used when introducing geometrical concepts (Barrantes, López and Fernández, 2015). In this sense, the effective application of different concepts representations in teaching supports the conceptual understanding development and potential success in complex problems solving.

Mathematics teaching-learning process in Cuba neglected mathematical processes such as content representations and its meaningful connections (Gamboa, 2022). Current improvement of the Cuban National Education System, consequently, presents guidelines related to mathematical skills. Logical-linguistic training is one of them, where work with standard representations of mathematical objects is prioritized. Pertinent connections stand as the principal challenge; so that, learning is made from a more genuine understanding, based on accumulated experiences. This scenario reveals the legitimate need for epistemological transformations to assume an interdisciplinary character.

The present research, focused on Cuban open curriculum, allows for concreteness in diverse contexts, complex realities and needs. Therefore, creative flexibility of participating teachers was considered.

They may decide the teaching objectives, curricular contents, development activities, proper evaluation and appropriate methodology that materializes in the classroom to establish interdisciplinary relationships. For example, if students are to assimilate a calculation procedure, it should be implemented comprehending its practical meanings. Assorted situations should be presented to move from one number representation form to another to communicate their observed results. It helps for collective reflection and effective integration of the worthy ideas contributed by all.

The explicit discussion on the topic highlights the essential need to broaden comprehensive perspectives regarding the complex nature of various academic disciplines in effective cooperation. Yu, Fan and Lin (2015) indicate context simulation is beneficial for improving students' skills establishing and analyzing questions, and then selecting and developing solutions. Meanwhile, Dreher and Kuntze (2015) emphasize that teachers' noticing the critical cases of dealing with multiple representations is based on both situated and global knowledge and viewpoints. Interdisciplinary relationships in mathematics are however significantly limited in these specific proposals.

Kazak, Wegerif and Fujita (2015) argue that dialogic theory contributes to the extensive analysis of understanding in mathematics classroom and emphasize the importance of emotional engagement. Like so, Bakker, Smit, and Wegerif (2015) claim the potential usefulness of dialogic scaffolding and effective teaching in mathematics education. In parallel, Leatham, Peterson, Stockero, and Van Zoest (2015) introduce mathematically meaningful pedagogical opportunities to develop on students' thinking. Similarly, Jones, Swan and Pollitt (2015) present comparative opinion as an assessment alternative based on collective expert judgments of students' work. However, a significantly limited look at content integration is revealed to consolidate students learning at the desired levels.

Learning must be placed in sociocultural scenarios of interpretation. Mathematical concepts are accompanied by practices in diverse situations. Therefore, it is necessary to extend this functional perspective to the meaningful relationship with other potential subjects with an interdisciplinary character.

Pantziara and Philippou (2015) present the direct relationship between affective constructs, including motivation, and student performance in mathematics. In addition, Darragh (2016) promotes cultural identity in mathematics education. Meanwhile, Takahashi and McDougal (2016) introduce collaborative lesson research to include successful institutional structures and practices that maximize their potential impact. These valuable proposals also require renewed guidelines to teach mathematics integrated to other disciplines potential and the school environment.

As a logical consequence, it is intended to improve teacher training with a new model for designing mathematics teaching-learning process, based on assorted mathematical content representations, their practical applications, and interdisciplinary relations. Teaching coherence is enhanced in such way, according to the productive work with interdisciplinary representation systems. This article presents evidence of that.

DEVELOPMENT.

Methodology.

The study was established according to experimentation (Cobb, Jackson and Dunlap, 2016), referring to the no need for alternative or control groups. The selected sample was compared before, during and after implementing the considered proposal. An explicit interpretive framework for making sense of teachers' actions was implemented. A coherent set of heuristics was also operationalized to design supports for their active learning. It was implemented in all specific pre-university education centers in Cuban National Education System (rural, urban, and vocational pre-university institutes).

The sample members were 121 qualified mathematics teachers from Las Tunas province. Independent municipalities (Puerto Padre, Jesús Menéndez, Manatí, Amancio Rodríguez, Jobabo, Colombia, Las Tunas and Majibacoa) were stratified in the empirical study.

An exploratory analysis was carried out monthly with selected teachers. This made it possible to contact them systematically. The measurement variable was teaching coherence, since a variable can also be the result of a process (Carballo and Guelmes, 2016). This is the articulation of interactions in the process, based on relationships between its components that are relevant and complement each other, so that there is meaningful interconnection that ensures its unity (Gamboa, 2020). In this respect, its operationalization presents observable and quantifiable relationships through the teaching coherence that is manifested as a variable resulting from the design of the mathematics teaching-learning process (Gamboa, 2020).

The statistical scale and the specialized software to evaluate mathematics teaching coherence (Gamboa, 2020) were taken as a specific reference in the present research. In this regard, significant volumes of relevant data were generated from the diverse evaluated teachers' actions. Interesting patterns of relevant and novel information were identified, based on finding existing relationships. This, more than to merely, gives a measurement value of the mathematics teaching coherence, was principally for searching possible causes in the behavioral patterns of observed data.

The statistical scale describes the specific criteria established to evaluate the appropriate level of execution of the key indicators and grant the corresponding categories. It allowed the functional evaluation of six precise dimensions, with 10 per capita indicators. Meantime, the specialized software was implemented to process more than 360,000 empirical data generated during the five academic years of active research. This made it possible to evaluate the teaching coherence of the measured sample, separately and all together. Therefore, it was possible to expose interesting structures in the data, understandable and useful patterns. Their considerable potential for generating predictive models, identifying hidden relationships, formulating valid conclusions and making reasonable decisions should be emphasized.

As a consequence, an ordinal scale was used in which each indicator showed a specific characteristic in mathematics teaching-learning process. There were six possible categories, that in qualitative terms, evaluated its execution level in the process for each measuring instrument (Table 1). In worthy addition, the continuity assumption of Gamboa (2018) was shared for its long amplitude. It is legitimate to analyze ordinal data parametrically, for its potential usefulness leading to fruitful results, as effective practice that receives the support of the academic community.

Table 1. Categories and criteria for assessing the level of performance of each key indicator.

Categories	**Criteria for assessing the level of execution and awarding the corresponding categories.**
Null (0)	There is no evidence of the presence of the key indicator in the process. Therefore, the level of accomplishment of the indicator is non-existent. This is the lowest category because it reveals the need to produce knowledge out of ignorance.
Poor (1)	There is evidence of the presence of the key indicator in the process. However, the execution is contrary to what is due, inadequately, in a harmful way. Against what is due, inadequately or improperly for the achievement of the integral formation of the students. It is harmful and damages this aspiration. It is preferable to demonstrate the awareness to do it, even if help is needed to do it well. This category reveals the need to bring out preceding knowledge, even if it is erroneous.
Fair (2)	There is evidence of the presence of the key indicator in the process. However, the execution is partially correct, not good. Medium or intermediate quality. It is between the categories Bad and Good. It contains some minor error. Similarly, if the key indicator is present, but is sufficiently undeveloped to produce an appreciable impact on the learners. This category reveals the need to investigate how incomplete and inaccurate knowledge becomes more complete and more accurate.
Good (3)	The usefulness and benefit of the presence of the key indicator in the process are demonstrated. The performance is demonstrated to maintain a positive value and is therefore worthy. It is performed properly, correctly, in an effective way. Without considerable inconvenience or difficulty. As needed, in an adequate manner for the achievement of the students integral training. In the manner deemed convenient, in accordance with this academic aspiration. Without antagonistic contradiction.
Very Good (4)	It is demonstrated that the presence of the key indicator in the process is extraordinarily good, that it excels in merit. It is demonstrated that the performance stands out for its optimal qualities. It is very good or excels in some quality with respect to the ordinary requirement. However, it does not create any product with which it defends its point of view, justifies its position and shows connections according to the organization of the subject. The level of execution stands out for its quality, for its superiority. It is highly valued and also distinguishes itself from the average.
Excellent (5)	It is evident the presence of the indicator in the process is extraordinarily good. In addition, depth and a creative level of execution are demonstrated. Some personal products to defend the point of view is created, showing maturity in the use or management of the indicator.

Corresponding to the above, selected indicators are considered as the primary units of proper valuation. The general categories determine the specific weight that each established criterion receives when evaluating them (Gamboa, 2020). Each criterion points out the key aspects to be considered within this empirical evaluation. It is significant to recognize the effective use of this established scale is engaged in an authentic assessment process. To evaluate teaching coherence, in a graduation from operational excellence to more inadequate levels, the specific ranges of categories were: Excellent ($4{,}75 < \bar{x} \leq 5$), Very Good ($3{,}75 < \bar{x} \leq 4{,}75$), Good ($2{,}75 < \bar{x} \leq 3{,}75$), Fair ($1{,}75 < \bar{x} \leq 2{,}75$), Poor ($0{,}75 < \bar{x} \leq 1{,}75$), Null ($\bar{x} \leq 0{,}75$). This scale was used both in the empirical research and in the experimental validation of the comprehensive proposal.

Teaching coherence for a chosen member of the selected sample is initially addressed. It is evaluated with the practical application of various convenient methods, specific techniques, and reliable instruments (M_n). The key indicator continuous measurement represents the historical average of the observed scores on the Likert scale used for it (Table 2). Averages (arithmetic mean) of the comprehensive evaluations in each indicator are performed ($I_i = \bar{x}(M_{1,i}; M_{n,i})$). These are used to procure the extensive evaluation of the specified dimensions, as well as the teaching coherence for those sampled ($\bar{x}(M_{1,1}; M_{n,60})$).

Table 2. Continuous measurement of teaching coherence for each chosen member of selected sample.

Var	Teaching coherence for each chosen member of the selected sample					
Dim	Objectives	Contents	Methods	Resources	Organization	Assessment
$\boldsymbol{M_1}$	$M_{1,1}, \ldots, M_{1,10}$	$M_{1,11}, \ldots, M_{1,20}$	$M_{1,21}, \ldots, M_{1,30}$	$M_{1,31}, \ldots, M_{1,40}$	$M_{1,41}, \ldots, M_{1,50}$	$M_{1,51}, \ldots, M_{1,60}$
$\boldsymbol{M_2}$	$M_{2,1}, \ldots, M_{2,10}$	$M_{2,11}, \ldots, M_{2,20}$	$M_{2,21}, \ldots, M_{2,30}$	$M_{2,31}, \ldots, M_{2,40}$	$M_{2,41}, \ldots, M_{2,50}$	$M_{2,51}, \ldots, M_{2,60}$
$\boldsymbol{M_3}$	$M_{3,1}, \ldots, M_{3,10}$	$M_{3,11}, \ldots, M_{3,20}$	$M_{3,21}, \ldots, M_{3,30}$	$M_{3,31}, \ldots, M_{3,40}$	$M_{3,41}, \ldots, M_{3,50}$	$M_{3,51}, \ldots, M_{3,60}$
$\boldsymbol{M_n}$	$M_{n,1}, \ldots, M_{n,10}$	$M_{n,11}, \ldots, M_{n,20}$	$M_{n,21}, \ldots, M_{n,30}$	$M_{n,31}, \ldots, M_{n,40}$	$M_{n,41}, \ldots, M_{n,50}$	$M_{n,51}, \ldots, M_{n,60}$
Ind	$I_1, \ldots, I_{10}$	$I_1, \ldots, I_{20}$	$I_{21}, \ldots, I_{30}$	$I_{31}, \ldots, I_{40}$	$I_{41}, \ldots, I_{50}$	$I_{51}, \ldots, I_{60}$
Dim	$\bar{x}(M_{1,1}; M_{n,10})$	$\bar{x}(M_{1,11}; M_{n,20})$	$\bar{x}(M_{1,21}; M_{n,30})$	$\bar{x}(M_{1,31}; M_{n,40})$	$\bar{x}(M_{1,41}; M_{n,50})$	$\bar{x}(M_{1,51}; M_{n,60})$
Var	$\bar{x}(M_{1,1}; M_{n,60})$					
$\boldsymbol{I_i = \bar{x}(M_{1,i}; M_{n,i})}$						

The teaching coherence for the overall selected sample (Table 3) is addressed later. This is calculated employing the previous averages of the key indicators for the sample members (N_n). In this sense, each indicator global average ($IG_i = \bar{x}(I_{1,i}; I_{n,i})$) provides the general conclusions of the variable (TC= $\bar{x}(I_{1,1}; I_{n,60})$).

Table 3. Continuous measurement of teaching coherence for the overall selected sample.

Var	Teaching coherence for the overall selected sample					
Dim	Objectives	Contents	Methods	Resources	Organization	Assessment
$\boldsymbol{N_1}$	$I_{1,1}, \dots, I_{1,10}$	$I_{1,11}, \dots, I_{1,20}$	$I_{1,21}, \dots, I_{1,30}$	$I_{1,31}, \dots, I_{1,40}$	$I_{1,41}, \dots, I_{1,50}$	$I_{1,51}, \dots, I_{1,60}$
$\boldsymbol{N_2}$	$I_{2,1}, \dots, I_{2,10}$	$I_{2,11}, \dots, I_{2,20}$	$I_{2,21}, \dots, I_{2,30}$	$I_{2,31}, \dots, I_{2,40}$	$I_{2,41}, \dots, I_{2,50}$	$I_{2,51}, \dots, I_{2,60}$
$\boldsymbol{N_3}$	$I_{3,1}, \dots, I_{3,10}$	$I_{3,11}, \dots, I_{3,20}$	$I_{3,21}, \dots, I_{3,30}$	$I_{3,31}, \dots, I_{3,40}$	$I_{3,41}, \dots, I_{3,50}$	$I_{3,51}, \dots, I_{3,60}$
$\boldsymbol{N_n}$	$I_{n,1}, \dots, I_{n,10}$	$I_{n,11}, \dots, I_{n,20}$	$I_{n,21}, \dots, I_{n,30}$	$I_{n,31}, \dots, I_{n,40}$	$I_{n,41}, \dots, I_{n,50}$	$I_{n,51}, \dots, I_{n,60}$
Ind	$IG_1, \dots, IG_{10}$	$IG_1, \dots, IG_{20}$	$IG_{21}, \dots, IG_{30}$	$IG_{31}, \dots, IG_{40}$	$IG_{41}, \dots, IG_{50}$	$IG_{51}, \dots, IG_{60}$
Dim	$\bar{x}(I_{1,1}; I_{n,10})$	$\bar{x}(I_{1,11}; I_{n,20})$	$\bar{x}(I_{1,21}; I_{n,30})$	$\bar{x}(I_{1,31}; I_{n,40})$	$\bar{x}(I_{1,41}; I_{n,50})$	$\bar{x}(I_{1,51}; I_{n,60})$
Var	$\bar{x}(I_{1,1}; I_{n,60})$					
$\boldsymbol{IG_i = \bar{x}(I_{1,i}; I_{n,i})}$						

The principal sources of needed information to investigate teaching coherence were present and former students, local teachers and administrators, educational work assistants, active parents, and community members. A perception scale was applied to them, and a printed sheet with the key indicators to measure the variable.

Methodological work, visit and departmental meetings reports were used. Student organization activities, official inspections, direct results of performance assessment and annual plans were implemented among additional products of the teaching process. Analysis and synthesis, induction and deduction were used as procedures of several methods, techniques, and instruments to examine the sources and collect the necessary data. In this way, the specific results were contrasted to comply with the statistical principle of not studying isolated facts.

The standard questionnaire, survey and interview were other instruments used in this exhaustive research. Specific problems comprehensive inventory, sentence completion and comparative study of the teaching-learning process authentic products were carried out. Simultaneously, direct observation of academic activities, formal and informal meetings with selected students, teachers and administrators were performed. They were implemented by making specialized, methodological and other support visits with a dialectical materialist approach.

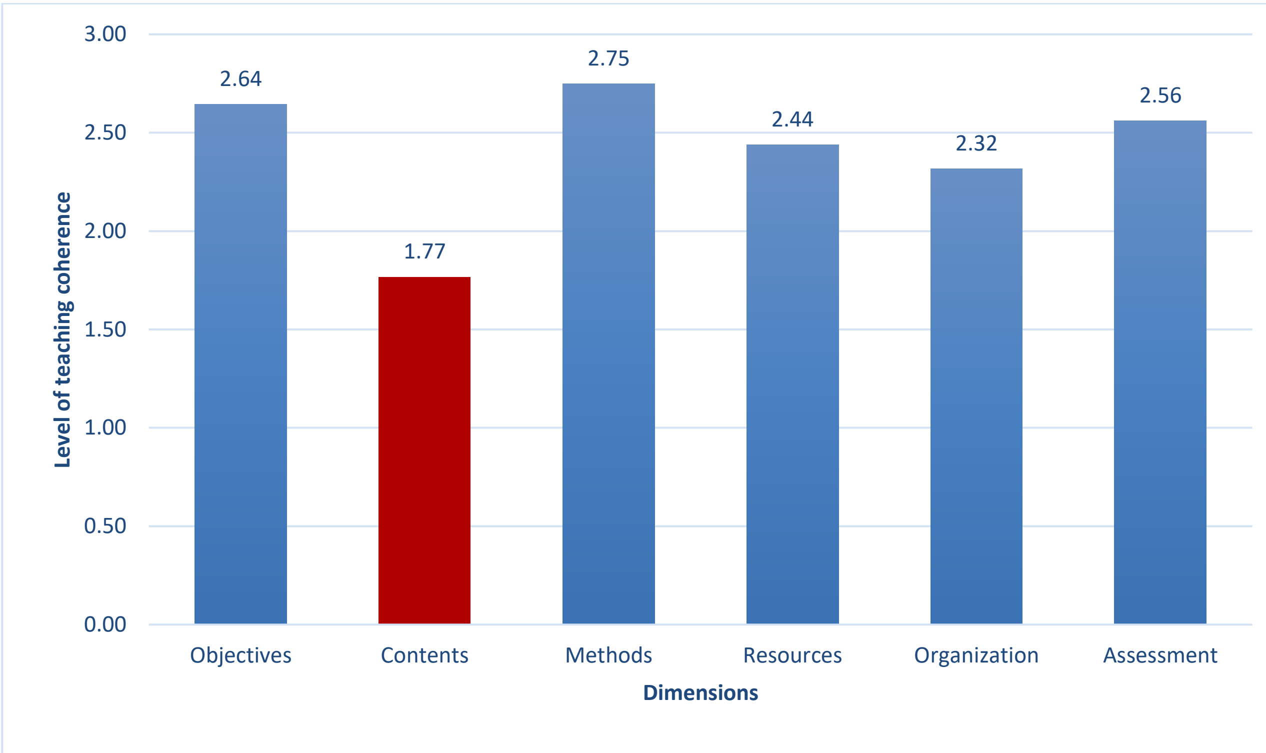
Results.

Initial teaching coherence characterization in Las Tunas province.

Typical students should be capable to select, use and produce representations to organize, document and communicate mathematical ideas to implement them in problems solving. Understanding a class situation implies having several adequate representations and being able to translate from one to another. This allows the exploratory study of various complex facets and fundamental properties of the same content, an interdisciplinary work in this search (Gamboa and Borrero, 2017). Rahmawati, Hidayanto, and Anwar (2017) present the legitimate need to identify the essential characteristics of the studied concepts to compare and interpret them. However, these specific proposals stay within the mathematical content logical framework and miss the limitless possibilities provided by the academic curriculum.

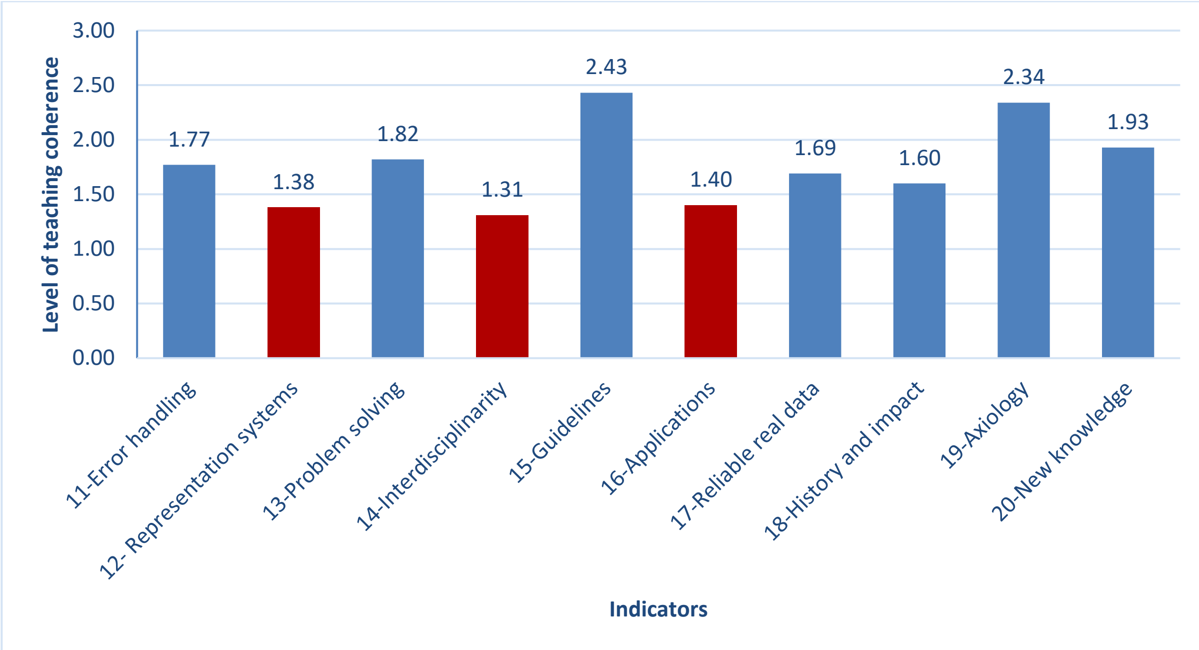
The synthesis of the global evaluation revealed the fair initial state of mathematics teaching coherence in Las Tunas province. In notable addition, the standard deviation of the empirical data introduced its dispersion and its heterogeneity. Figure 1 shows that no specified dimension reached an established category higher than fair, although academic contents stand for the one with the severest difficulties. Therefore, it was a critical need to improve the initial arbitrary selection and controversial organization of cultural and experiential contents, in marked relation with their scientific value. This, initially, became the potential cause of the inadequate levels of development of the students' productive capacities. This affected the students' awareness for logical reasoning, symbolization, intellectual abstraction, academic rigor and sufficient precision that distinguish mathematical thinking.

Figure 1. Initial state of teaching coherence per dimensions.



The consistent patterns of specific behavior in the content dimension data (Figure 2) revealed the standard deviation was higher than the others, with notable differences to be addressed. Methodological problems in relating explicit meanings to interdisciplinary content were the most significant. Mathematical knowledge is interpreted in remarkable uniformity with its internal logic. However, contacts with other curricular contents are lost. Therefore, it was revealed as the principal cause of insufficiencies in the rest of the key indicators. Furthermore, there were manifest difficulties to implement representation systems according to the complex students' reality. This discouraged teaching potential practical applications for solving context genuine problems.

Figure 2. Initial state of teaching coherence in contents dimension.



Effective teachers "must also be able to make sense of students' mathematical thinking, understand the organization and intent of curricular materials, and select contexts to motivate and highlight mathematical ideas" (Patterson, Parrott and Belnap, 2020, p.807). This makes explicit the fundamental need to present assessments focused on implementing representations, building arguments, and selecting problems to bring out important ideas. This should be accomplished according to the teaching-learning contexts.

The effective presentation of mathematical teaching situations linked to several disciplines was resented. Overworked teachers failed to enhance the students' development towards higher levels of cognitive performance, through complex tasks of interdisciplinary nature. It was limited the extensive planning of connecting academic contents to practical applications, modeling diverse situations, and investigating sufficient regularities or consistent patterns of complex reality. Simultaneously, the logical coherence of the conceptual system was neglected, so that the diverse representations would complement each other as a mathematical process. This would add noticeable peculiarities that would increase teachers' genuine commitment and students' active participation for their meaningful learning.

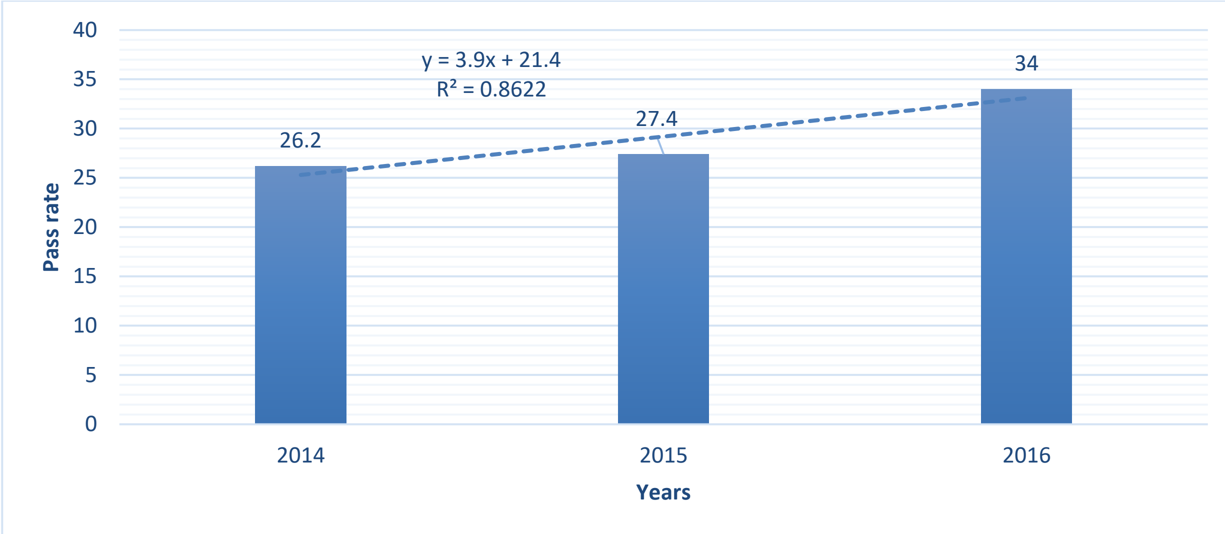
The potential problems determination was vague. Several teachers took into account imprecise specific cognitive tasks during the new content gradual assimilation process. It was evident in the deficient instruments elaborated for formative and summative assessment. These methodological shortcomings indicated significant variances in the underprepared students’ results. They showed promising results in teacher’s direct examination, but inevitably failed in external evaluations (provincial and national).

As a logical consequence, this had a negative impact on exposed students learning during the three academic years of this initial characterization. The tenth and eleventh graders provincial mathematics examinations results showed that more than 600 failed the subject each school year. At the same time, the twelfth graders national examinations results did not exceed 90.0 % of passing rates.

A deeper study was performed on the key indicators identified as the principal causes of insufficiencies. It was confirmed that students' negative results were even more worrisome. The tenth and eleventh grade provincial tests showed that students’ accurate answers did not exceed 35% in diverse format questions (

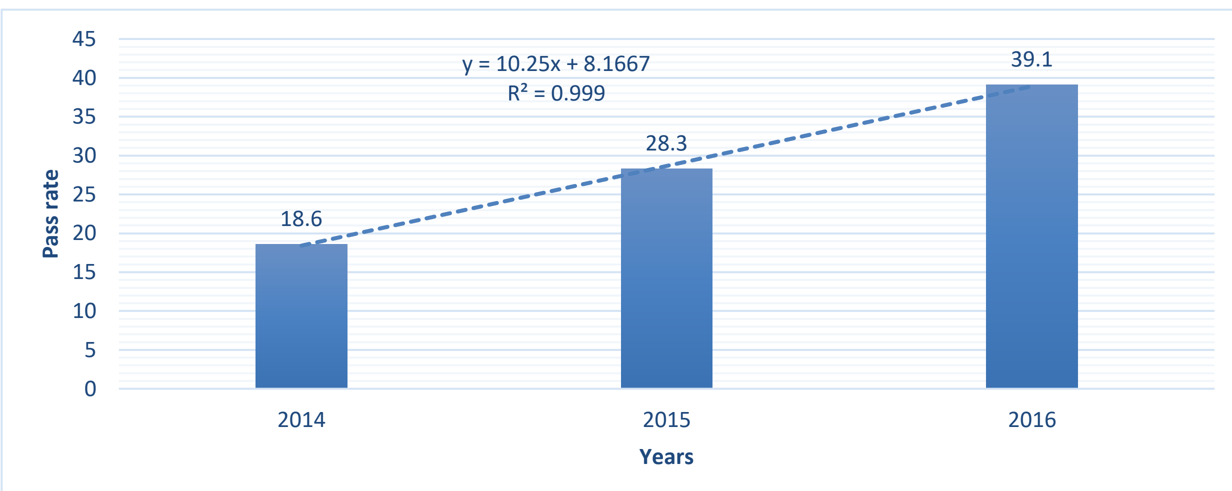
Figure 3). These critical questions evaluate the fundamental understanding of conceptual system meanings.

Figure 3. Pass rate of tenth and eleventh graders in diverse format questions of provincial tests per year.



The twelfth-grade national examinations for selective admission to Higher Education indicated a similar behavior. It was confirmed that students' accurate answers did not exceed 40% in interdisciplinary questions (Figure 4).

Figure 4. Pass rate of twelfth graders in interdisciplinary questions of national examinations for selective admission to Higher Education per year.



Likewise, a comprehensive study was performed by independent municipalities, specific type of academic institution and grade (10th, 11th, 12th) to establish sufficient regularities. This revealed a scenario far from the Cuban educational aspirations. As a necessary consequence, it was revealed the urgent need to implement actions aimed at improving the mathematics teaching-learning process.

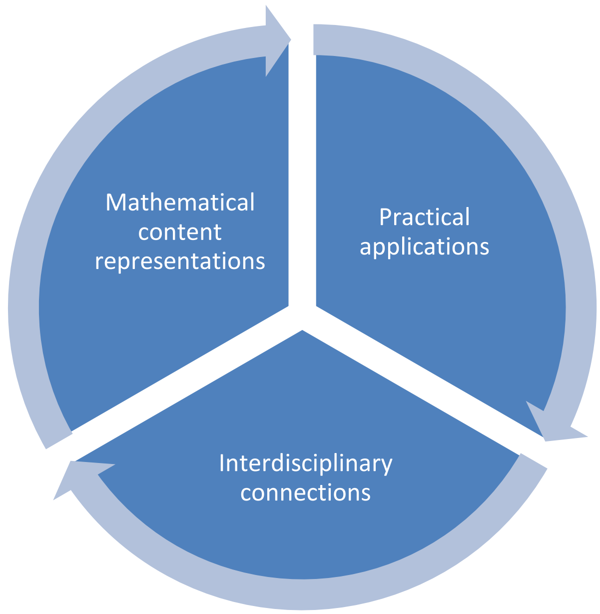
Teachers should relate the various meanings of mathematical contents with interdisciplinary applications.

Proposal.

A dynamic system containing two subsystems with subordination and complementary relationships is proposed. The first one acquires an organizing function according to the contextual reality. It pursues contextual consistency with complementarity relations of the mathematical contents' representations, their practical applications and the interdisciplinary connections. The second one undertakes a planning function in direct correspondence with the preceding organization. Therefore, it is subordinate to the first one. It aims at interdisciplinary coherence with complementary relationships of the didactic studies to be carried out (mathematical content, cognitive, instructional, and academic performance).

The organizing subsystem (Figure 5) requests the identification of the mathematical content representations. This is essential for students' learning to develop in a meaningful way. It promotes the possible creation and logical development of new ones as cognitive progress sign. Practical applications provide the potential use of the academic content studied in everyday life. Interdisciplinary connections allow meaningful relationships that students must be able to recognize, understand, apply, analyze, evaluate, and create to produce coherent ideas within and outside the mathematical context. In this viable way, a desired socialization can be generated that leads to greater coherence in educational influences. It generates a research process that enhances access to valuable content, useful in decision making for proper planning at various exploratory stages. The principal aim is to have enough organized information to develop interdisciplinary representation systems in the following subsystem.

Figure 5. The organizing subsystem.

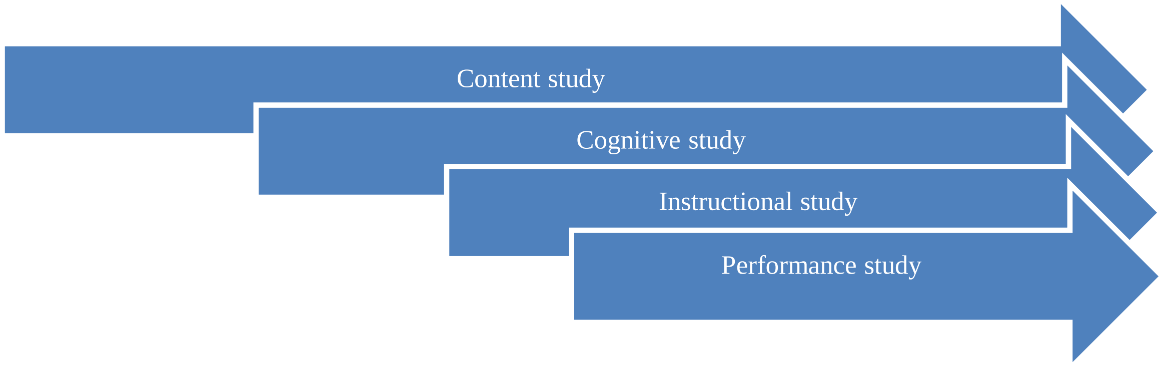


The effective teaching of mathematics allows the extensive study of fundamental facts, specific phenomena, and complex processes in their multiple interrelations, considering interdisciplinary connections. Mathematical contents are active enhancers of dependence and complementarity relationships with other disciplines. This helps students to recognize the potential usefulness and proper meaning of what they learn. Mathematics contributes to the necessary determination of mutual interdependence, convergence, and complementarity relationships among the academic disciplines of the official school curriculum.

Castro and Rico (2021) refer to didactic analysis from the specific content. Competent teachers identify and organize the meanings diversity, representation system, conceptual structure, and phenomenology. However, this must be complemented from an interdisciplinary character. This provides more elevated levels of contextualization, interpretation and complex problem solving. The great value of organic synthesis is underlined with the remarkable contextual peculiarities.

The planning subsystem (Figure 6) establishes the teaching-learning process with interdisciplinary character. This allows the needed resolution of academic exercises, practical problems, and meaningful activities of complex reality. Reliable data are integrated, and essential facts are presented. Consistent patterns are sought, and reasonable conjectures are made. Consequently, integrative frameworks are established, and cross-disciplinary cooperation takes place.

Figure 6. The planning subsystem.



The content study focuses on the mathematical topic involving an interdisciplinary approach. This specific proposal promotes critical and reflective thinking. Likewise, the mathematical knowledge system is linked to other disciplines and a distinctive context. Active teachers must identify, select, and organize the expanded meanings of mathematical topics.

Two representations categories were taken for this specific research. These are intrinsic and extrinsic. The intrinsic ones are constructed in each independent person's active mind in a particular way. Meantime, the extrinsic ones allow their possible interaction, conscious perception, and deliberate manipulation. Both have a systemic character and influence each other in cognitive activity (Macías, 2016). This reveals the dynamic and flexible aspect of mathematical thinking processes, where students imprint their personal characteristics on problems solving. However, it is expected to broaden this holistic perspective based on the legitimate need to know how to solve interdisciplinary problems. Content knowledge must be problematized and placed in the complex environment of the learner's active life. This requires a shared redesign, which orients and structures the school mathematical discourse (Cantoral, 2018). This is valuable to address the knowledge nature complexity and its successful operation at cognitive, didactic, epistemological, and social levels.

The previous subsystem prepared favorable conditions for not working with a unique representation of the academic content to be learned. It organized the complex scenario so as not to employ several in parallel, assuming that students will be able to establish successful relationships on their own. Effective teachers must be able to relate parallel representations and integrate them to develop interdisciplinary representational systems. Potential complexity and cognitive mobilization involved in this constructive way make it possible to move from one proper representation to another. Students' cognitive ability to recognize essential meanings to establish productive interdisciplinary connections and transfer knowledge meaningfully enhances mathematical understanding.

Typical students mobilize diverse representations, and it is necessary to carefully coordinate them. Extensive exploration of possible solutions to potential problems is needed. Modeling reality, effective communication, logical justification, and gradual establishment of necessary connections between varied mathematical objects and other disciplines are equally demanded. These are interrelated; they cannot be instantly comprehended in prolonged isolation from those used in other disciplines on which they are scientifically based and applied. Starting from the integrative content of interdisciplinary representation systems generates relevant information for the practical solution of mathematical problems.

Cognitive study focuses on learning. Teachers describe their possible hypotheses about students' academic progress in content assimilation when faced with interdisciplinary tasks. This allows teachers to state and organize the specific skills they expect students to develop in the mathematical subject with a marked interdisciplinary character. So, conjectures are made about the continuous progress in the gradual assimilation of knowledge, skills, and ways of acting to solve tasks linked to other contexts. This implies, among other key aspects, describing and relating the necessary capabilities that students have prior to direct instruction as well as their valuable contribution to competencies growth. Typical students progressively achieve a deeper mastery and are prepared to experience more integrative problems related to frequent mathematical topics of an interdisciplinary nature.

The following step is to explore the available resources to achieve the specific goals in the instructional study. So, the prime focus will be on teaching. Teachers must carefully select, or design exercises, potential problems or productive activities where necessary knowledge is properly integrated with diverse areas of mathematics itself and with other disciplines. This favors the gradual development of students towards more significant levels of cognitive performance, through increasingly complex tasks. To move from excessive dependence to cognitive independence and genuine creativity is the ultimate aim. It is essential to study how to encourage conscious reflection, meanings analysis and contents representation. Therefore, it is noteworthy the continuous systematization of their academic knowledge, cognitive skills, and proper attitudes.

The instructional study is revealed as the elaborate description of the specific activities to be tentatively proposed to the students. This is taking into account the possible variety of task types that typically emerge from the content study, the specific needs of the students and the available resources. Structured description of the mathematical topic on which the instructional activity relies is required. It should be performed from its conceptual structure, its representation systems, its phenomenological analysis, and its modeling possibilities. Such way, teachers consider interdisciplinary criteria to study, select and design integrative tasks. Like so, diverse scientific areas are linked to overcome interdisciplinary challenges.

The performance study completes a cycle in this planning subsystem, focusing on what will happen when the mathematics teaching-learning process is carried out. Guiding principles and standard procedures of authentic assessment will serve as the fundamental basis for doing so. Potential capabilities that students have developed and the possible difficulties that they may have shown up to that precise moment due to the interdisciplinary activities will be determined. This information feeds into a new cycle, where the key focus will be on comprehensive assessment, centering on monitoring students learning.

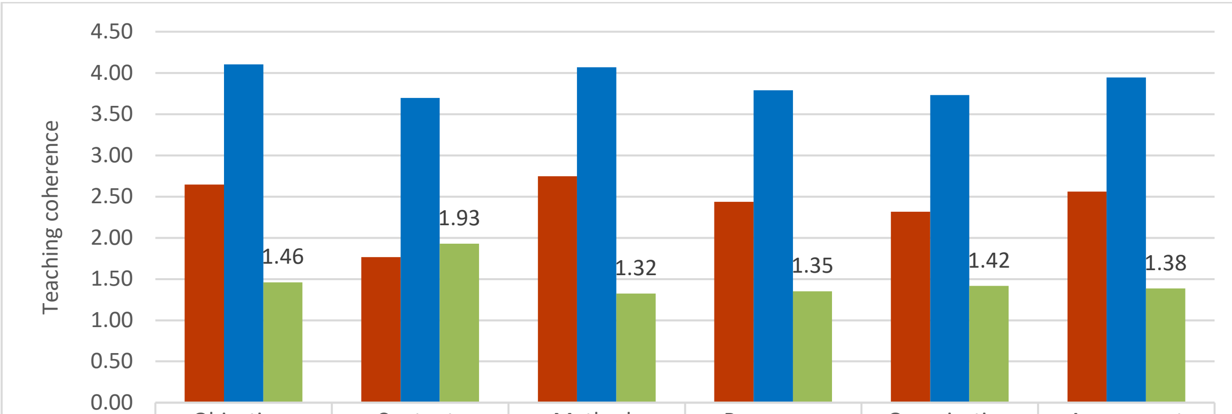
Interdisciplinary coherence represents the essential quality attempted in the second subsystem. It is identified with the gradual integration and continuous interaction of meaningful connections, based on interdisciplinary representation systems of the teaching contents. Intentionally allow the genuine transformation of the general essence into the singular and the counter movement. This favors the teaching-learning process according to educational objectives and the agents involved can discover the global meanings. In summary, the specific model presented here reasonably achieves teaching coherence based on contextual and interdisciplinary aspects.

Discussion.

Reflective knowledge is claimed in the authentic assessment of mathematics applications. It recognizes the need for mathematics serving specific objectives linked to everyday life. The principal difficulty of the authors studied lies in the disciplinary stance. It is demanded a more integrative view of the interdisciplinary mathematics teaching potential. Selected sample initial status confirms this.

The specific model proposed as a direct result of this empirical research focuses on addressing the previous situation. In this section, an effective movement from the initial status to the desired one is revealed. Indeed, the superior reliability of the central tendency measures was demonstrated, as the processed data standard deviation was reduced. Notable progress was achieved in all determined dimensions (Figure 7). Academic contents, organizational forms and teaching aids had the lowest coherence in the ultimate status, but they initially encountered the severest difficulties. As shown, when transformed, they favorably influenced the visible improvement and gradual consolidation of the rest. Content, as the identified cause, precisely represented the dimension with the most substantial growth. Consistent growth of the measurable objectives was likewise significant. This properly indicated the comprehensive understanding of the teaching coherence need by those sampled.

Figure 7. Growth of teaching coherence per dimension.



	Objectives	Contents	Methods	Resources	Organization	Assessment
Initial	2.65	1.77	2.75	2.44	2.32	2.56
Final	4.11	3.70	4.07	3.79	3.73	3.94
Growth	1.46	1.93	1.32	1.35	1.42	1.38

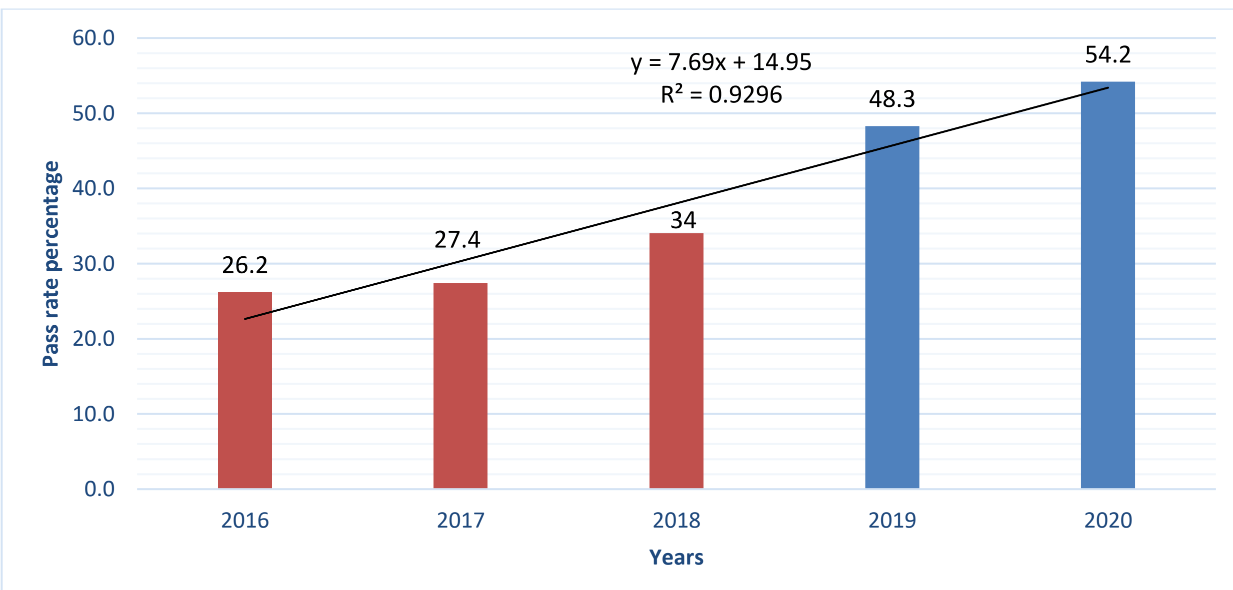
The meaningful relationships adequately established between the instructional, educational, and developmental aspects of teaching mathematics were significantly increased. Representation systems adequate to the skills structure, their potential relationships, personal limitations, and potentialities according to the complex reality of those involved were implemented. In worthy addition, practical meanings were more frequently related to interdisciplinary contents. Likewise, contemporary knowledge was enthusiastically promoted answering to the creative activity experiences system with appropriate teaching materials and available technological resources.

Several academic researchers have typically presented the cognitive process of translating mathematical representation from verbal to graphic, through others such as symbolic, schematic, equations, numerical. In this particular sense, the intellectual capacity to produce this accurate translation precisely represents a fundamental ability to properly build conceptual and mathematical thinking (Haji, 2019).

Adequate representations are evidence of thinking. There is in odd contrast a marked tendency to just present their extensive diversity in comparative isolation. Then, disconnected thinking is indeed stimulated. Therefore, there is a legitimate need to carefully attend to this mathematical process with the required interdisciplinary as an added value.

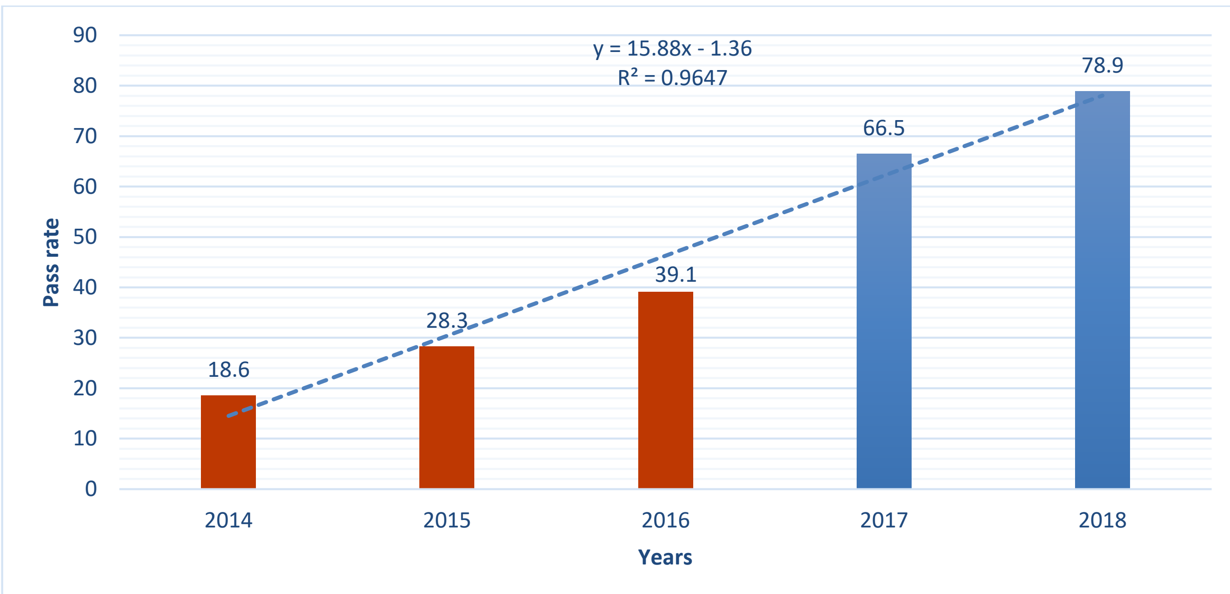
Local students' necessary knowledge experienced academic progress with the experimental model implemented. Mental operations were consolidated, such as: analyzing and synthesizing, comparing, and classifying, generalizing and concretizing, abstracting and particularizing. Results in various formats questions that evaluated the conceptual system coherence in tenth and eleventh grades provincial examinations were satisfactory (Figure 8). There was a gradual growth of 28% and the continuous initial trend line slope was almost doubled.

Figure 8. Results in various formats questions that evaluated the conceptual system coherence in tenth and eleventh grades provincial examinations per year.



Las Tunas exhibited an upward trend in the Higher Education national entrance examinations (Figure 9). The direct impact of the students' training influenced the recognized fact that more than 89% of the enrollment per academic year completed these specific tests, even though they were not mandatory. Likewise, for the first time, the pass rate exceeded 90%, consecutively in the two years of the proposal validation. This province continued above the national average. It produced an appreciable social impact on the local human resources development, professional life projects and territorial scientific-technical improvement. Results in interdisciplinary questions were also satisfactory. There was a gradual growth of 60.3% and the continuous initial trend line slope was multiplied by more than 1.5.

Figure 9. Results in interdisciplinary questions of the Higher Education national entrance examinations.



Interdisciplinary character allows optimizing the continuous process of argumentations and necessary operations with mathematical concepts. An added value is achieved in the effective transfer from one specific form of mathematical object representation to another. Proper possible models' interpretation is favored, as well as the frequent use of appropriate resources for the mental and practical rationalization of the work. This imposes the genuine need to link learning with students' experiential situations, for the logical understanding of the academic content. However, it facilitates directing the cognitive process to the explicit formulation and ultimate resolution of contextual complex problems. Likewise, this also makes it possible to deepen the scientific nature of the intended content and the meaningful relationship between prevailing theory and current practice. The key idea is additionally to enhance the recombination of scientific work by integrating data, posing questions, interacting with facts, validating hypotheses and drawing conclusions. It includes potential links between ways of thinking, feeling and acting to combine mathematics knowledge systems with other disciplines.

CONCLUSIONS.

An effective model is presented to enhance the necessary connections between the extensive diversity of mathematical contents representations, their practical applications, and interdisciplinary relations. These three key indicators represent the principal causes of teaching coherence insufficiencies. The complementary relationships among them are essential for organizing and planning the mathematics teaching-learning process.

The specific model practical implications reveal its potential relevance to the productive work with interdisciplinary representation systems. This favors integrative frameworks and interdisciplinary cooperation for the necessary resolution of academic exercises, practical problems, and significant activities of complex reality.

The empirical research revealed evident limitations in active teachers and school administrators' preparation. There is a genuine need to progressively expand sufficient training with the necessary interdisciplinary collaboration beyond the typical classroom in school-family-community effective cooperation. This precisely identified a necessary approach for further research, to experience diverse mathematics contexts.

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