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TÍTULO: Comparación de métodos iterativos con orden de convergencia tres y cuatro.

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RESUMEN: En el artículo realizamos experimentos con cuatro métodos iterativos para encontrar la raíz real de una ecuación no lineal. Tres de los métodos están en orden de convergencia 3 y uno de orden de convergencia 4. Dos de los métodos requieren el cálculo de una segunda derivada. Entonces, todos los métodos que se comparan son una modificación del método de Newton, e imponemos las condiciones de tipo de Newton para la aproximación inicial.

PALABRAS CLAVES: Ecuación no lineal, métodos iterativos, aproximación inicial.

TITLE: Comparison of Iterative Methods with order of Convergence Three and Four.

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ABSTRACT: In the article, we conduct experiments with four iterative methods to find the real root of a non-linear equation. Three of the methods are in order of convergence 3 and one of order of convergence 4. Two of the methods require calculation of a second derivative. So, all the methods being compared are a modification of the Newton's method, we impose Newton's type conditions for the initial approximation.

KEY WORDS: nonlinear equation, iterative methods, initial approximation.

INTRODUCTION.

Numerous methods are used to solve linear and non-linear equations that are difficult to solve using analytical methods. Through these methods, the solution is obtained as a boundary of descending rows, the members of which are computed by the same iteration formulas. We are looking for a simple root of a non-linear equation $f(x) = 0$, where $f : I \subset R \to R$ for open interval I. The solution is in two stages - first the roots are located, then the roots are found. To locate the roots, we are looking for a closed interval (a,b), in the edges of which the function has different characters $f(a).f(b) < 0$, which guarantees the presence of at least one root in this interval. Iterative formulas are used to specify the roots, with Newton's formula being the most popular and preferred:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad . \qquad (1)$$

In approximating the Newton's definite integral is approximated by use of different techniques

$$f\ (x) = f\ (x_n) + \int_{x_n}^{x} f'(t)\ dt. \qquad (2)$$

Many authors have received new methods, modifications of the Newton formula. In order to improve of convergence of lines and the number of iterations, many authors have modified Newton's formula and developed various iterative scheme using a variety of techniques; for example, Chun, offers an iteration formula involving a new weight function (Chun, 2005).

By approximating the indefinite integral by a Simpson's formula, Frontini and Sormani offer an iterative scheme with order of convergence 3 (Frontini and Sormani, 2004), and Noor using the trapezoid formula receive another modification of the Newton method (Noor, 2010; Nazoktabar & Tohidi, 2014). Another approach is used by Homeier (Homeier, 2004; Selvanayaki, 2017), who uses the inverse function $x = f\ (y)$, instead of $y = f\ (x)$ from the Newton's theorem and suggests modification order of convergence 3. For finding new iterative methods using techniques such as quadrature formulas / Newton- Kouts/, homotopy – described in details by Noor (Chun, 2007) and other techniques of decomposition, as a result of witch methods with cubic and quadratic order of convergence are obtained (Chun, 2005; Hasanov, e al. 2002; Frontini and Sormani, 2004; Ribera, e al. 2008). The methods we compare in the article are modifications of the Newton's method: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$. (3)

The authors of the methods have performed experiments with different functions to find the root, compared to the number of iterations, order of convergence, and computational accuracy. Experiments have been made with initial approximations for which there are not placed conditions. Since each of them is a modification of the Newton's method, using different techniques, we examine and compare these methods by selecting the initial approximations, which satisfy the following conditions:

1. We explore the functions in the interval (a, b), for which $f(a).f(b) < 0$, which ensures a root in the corresponding interval.

2. We require in the corresponding interval, the first and the second derivative to be continuous for $\forall x \in (a, b)$:

$f'(x) \neq 0 \quad and \quad f''(x) \neq 0.$

3. For initial approximation we use this end of the interval$x_0 = a \; or \; x_0 = b$, for which $f(x_0).f''(x_0) > 0$.

These imposed conditions, constrict the interval in which we can explore the function and limit our choice of initial approximation. Our aim is to analyze the behavior of methods when Newton's type restrictions are imposed and to distinguish the method with the fastest convergence. With the help of computer program MATLAB, we calculate the order of convergence and the iterations required of each method for finding the root, contained in the selected interval and compare the selected methods.

Description of the algorithms compared in the article.

Method 1. Farooq Ahmed's method, described in article (Farooq, 2014).

Another technique for obtaining iterative circuits is the modified homotopic perturbation technique used in article (Farooq, 2014) by Farooq Ahmed. He offers a class of iterative schemes with order of convergence 4 and higher. In our article we use one of these iterative formulas for comparison with the methods described below. In article **(**Farooq, 2014**)**, the author considers a non-linear equation

$$f(x) = 0. \qquad (4)$$

Assume that r is a simple root for nonlinear equation (4), and γ is an initial approximation close enough to r. The author inserts function the auxiliary function (x), such that

$$f(x).g(x) = 0. \qquad (5)$$

He rewrites the nonlinear equation (5) as a system of coupled equations using the Taylor series technique.

After a number of considerations described in (Farooq, 2014), he reaches new iterative methods involving the auxiliary function $g(x_n)$. We focus on algorithm 7, described in detail in the article. Function $g(x_n)$ is a constant.

Algorithm 7:

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = y_n - \frac{2f(y_n)}{f'(x_n)} + \frac{f(y_n).f'(y_n)}{[f'(x_n)]^2} - \frac{1}{2}\frac{f(y_n)^2}{[f'(x_n)]^2}.\frac{f''(y_n)}{f'(x_n)} \quad . \qquad (6)$$

The next theorem proves the convergence of methods. The theorem is proved in article (Farooq, 2014).

Theorem 1.

Assume that the function f : D ⊂ R → Rfor an openinterval in Dwith simplerootr ∈ D. Let f(x) be a smooth sufficiently in some neighborhood of root and then Algorithms 7 one sees that it has fourth-order convergence.

Method 2. Halley's method, described in article (Homeier, 2005).

According to Traub (Selvanayaki, 2017), Halley's method is one of the rediscovered and studied iterative method in the history of mathematics. The of Halley's method owns cubic convergence in the approximation of simple zeros, but multiple zeros convergence is linear. The iteration formula proposed by Halley:

$$x_{n+1} = x_n - \left(\frac{f'(x_n)}{f(x_n)} - \frac{1}{2}.\frac{f''(x_n)}{f'(x_n)}\right)^{-1} . \qquad (7)$$

Obreshkov issued the following modification of the method of Halley for zeros of multiplicity m. Cubic convergence remains with this modification, but to apply this method, we need to know the multiplicity m.

$$x_{n+1} = x_n - \left(\frac{m+1}{2m}.\frac{f'(x_n)}{f(x_n)} - \frac{1}{2}.\frac{f''(x_n)}{f'(x_n)}\right)^{-1}.$$

Form=1 have (11)

Another modification of the Halley's method is the iterative method of approximation of multiple zeros proposed. This method has quadratic convergence and requires no prior knowledge of the multiplicity of zero

$$x_{n+1} = x_n - \left(\frac{f'(x_n)}{f(x_n)} - \frac{f''(x_n)}{f'(x_n)}\right)^{-1} . \qquad (8)$$

The next theorem proves the convergence Halley's method with iterative scheme (8).

Theorem 2.

We assume that $f \epsilon C^3(a,b)$and there is a number$x \in (a,b)$, where$f(x) = 0$. If $f'(x) \neq 0$, then there is $\delta > 0$ such that the sequence $\{x_n\}_{n=0}^{\infty}$ defined by the iterative scheme (8) for $n = 0,1, \ldots$

will be closer tox for any initial approximation $x_0 \in (x-\delta, x+\delta)$. In addition, if x is a simple root, then $\{x_{n+1}\}$ there will be a order of convergence $R = 3$, i. e

$$\lim_{n\to\infty} \frac{|x-x_{n+1}|}{|x-x_n|^3} = \lim_{n\to\infty} \frac{|E_{n+1}|}{|E_n|^3} = A.$$

Method 3. Homeier‘s method, described in articles (Homeier, 2004; Selvanayaki, 2017).

The idea of Homeier again based on Newton's method, but instead of the function $y = f(x)$ the reverse function $x(y)$ is used $x(y) = x(y_n) + \int_{y_n}^{y} x'(\eta)\, d\eta$.

Thus, a class of Newton's cubic methods is obtained, the most efficient of which is:

$$x_{n+1} = x_n - \frac{f(x_n)}{2}.\left(\frac{1}{f'(x_n)} + \frac{1}{f'\left(x_n - \frac{f(x_n)}{f'(x_n)}\right)}\right) . \qquad (9)$$

The next theorem proves the convergence of the methods. The theorem is prooved in (Selvanayaki, 2017).

Theorem 3.

Let ω_γ and τ_γ for γ =1. . ., m be the weights and abscissas, resp., of an m-point interpolar quadrature for the interval (0, 1), that is at least of order1. Letf : R → Rbe a smooth function witha simple zerox*and abbreviate the scaled derivatives off at the zero by

$$C_k = \frac{f^k(x_*)}{f'(x_*)}.$$

Then, the iterative scheme

$$x_{n+1} = x_n - f(x_n) \sum_{\gamma=1}^{m} \frac{\omega_\gamma}{f'\left(x_n - \frac{\tau_\gamma f(x_n)}{f'(x_n)}\right)}$$

converges cubically to x_* in a neighborhood of x_*. The errors $\varepsilon_n = x_n - x_*$obey the order relation

$$\varepsilon_{n+1} = \frac{K}{3!}\varepsilon_n^3 + O(\varepsilon_n^4) \text{ за } \varepsilon_n \to 0 .$$

Here K is the constant

$$K = 3C_2^2 - C_3 + 3\sum_{\gamma=1}^{m} \omega_\gamma \tau_\gamma^2 (C_3 - 2C_2^2).$$

Form $= 2$, $\omega_1 = \omega_2 = \frac{1}{2}$, $\tau_1 = 1 - \tau_2 = 0$, we obtain scheme (9) with $e_{n+1} = \frac{C_3}{12} e_n^3 + O(e_n^4)$.

Method 4. Kou's method, described in article (Chun, 2007).

In this method, the Newton's integral is solved within new boundaries

$$f(x) = f(y_n) + \int_{y_n}^{x} f'(t)\,dt,$$

where

$$y_n = x_n + \frac{f(x_n)}{f'(x_n)}.$$

The rule for the middle point is used to solve the integral

$$\int_{y_n}^{x} f'(t)\,dt = (x - y_n).f'\left(\frac{x+y_n}{2}\right).$$

And for$f(x) = 0$ a new method is proposed

$$x_{n+1} = y_n - \frac{f(y_n)}{\frac{f'(x_{n+1}^* + y_n)}{2}},$$

where

$$x_{n+1}^* = y_n - \frac{f(y_n)}{f'(x_n)}$$

$$y_n = x_n + \frac{f(x_n)}{f'(x_n)}$$

And then,

$$x_{n+1} = x_n - \frac{f\left(x_n + \frac{f(x_n)}{f'(x_n)}\right) - f(x_n)}{f'(x_n)}. \qquad (10)$$

The next theorem proves the convergence of the methods. The theorem is proved in (Chun, 2007).

Theorem 4.

Assume that the function $f : D \subset R \to R$ has a simple root $x^* \in D$, where D is an open interval. If $f(x)$ has first, second and third derivatives in the interval D, then the method defined by (10) converges cubically to x^* in a neighborhood of x^*.

The acceptable error is:

$$e_{n+1} = \frac{1}{2}.\frac{f''(x_n)^2}{f'(x_n)^2}.e_n^3 - \frac{1}{3}.\frac{f^{(3)}(x_n)}{f'(x_n)}.e_n^3 + O(e_n^4).$$

Numerical experiments.

In tables 1 to 5 the experiments made by the above methods are presented. We stop our attention at five functions, that have continuous first and second derivative to the corresponding interval (a, b), in which we look for the root. The initial approximation x_0, is the end of interval (a, b), for which$f(x_0).f''(x_0) > 0$.

We use the following test functions and display the computed results:

$f_1(x) = 0.5.e^x - 5x + 2,\ x_n = 3.401795803e + 00$

$f_2(x) = e^x - 4x^2, x_n = 4.306584e + 00$

$f_3(x) = x^2 - e^{-x} - 3x + 2,\ x_n = -2.99223e + 00$

$f_4(x) = x.\exp(x) - 1,\ x_n = 567.143290e - 003$

$f_5(x) = (x + 2)e^x - 1,\ x_n = -4.428544e - 01$

All calculations were made using a computer program Matlab 7.6.0. For the stop criterion, we use the difference between the last two approximations δ to be less than tol = 10e-15.

As convergence criterion used 2 conditions:

- The absolute value of the function from the approximated root found must be less than the specified accuracy $|f(x_n)| < tol$.
- The absolute value of the difference between the latter and the penultimate approximation –

$|x_n - x_{n-1}| < tol.$

Definition 1.

Letα be a root of the function f(x) and suppose that $x_{n-1}, x_n, x_{n+1}$are three consecutive iterations closer to the root α. Then the computational order of convergenceρ can be computed using the formula:

$\rho = \frac{\ln(|x_{n+1}-\alpha|/|x_n-\alpha|)}{\ln(|x_n-\alpha|/|x_{n-1}-\alpha|)}$.

Table Description.

We will use the following labels for the methods with which we do experiments:

M1 – Algorithm 7 (6) described in (Farooq, 2014).

Halley – Halley's method (7) described in (Homeier, 2005).

Homeier – Homeier's method (7) described in (Weerakoon and Fernando, 2000; Noor, 2010).

Kou – Kou's method (9) described in (Chun, 2007).

1. It–number of iterations.

2. $|f(x_n)|$ – the absolute value of the function in a point $x = x_n$- last approximation.

3. δ – difference between the last two approximations.

4. ρ – the order of convergence.

Function 1. We consider the nonlinear equation $f_1(x) = 0.5.e^x - 5x + 2$ in the interval, where the root is $x_n = 3.401795803e + 00$. We scroll the initial approximations to the right of the root, where the graph of the function and of its second derivative pass over the Ox axis and accept a positive sign.

Table 1.

$f_1(x) = 0.5.e^x - 5x + 2$	$x_n = 3.401795803e + 00$				10e-15
		it	$\lvert f(x_n)\rvert$	δ	ρ
Homeier	$x_0 = 5$	4	3.55e-15	8.92e-08	3.05e+00
Kou		5	0.00e+00	7.11e-15	3.00e+00
Halley		4	0.00e+00	4.29e-07	2.90e+00
M2		4	0.00e+00	5.77e-07	3.59e+00
Homeier	$x_0 = 7$	5	3.55e-15	2.03e-06	3.05e+00
Kou		6	0.00e+00	5.34e-11	2.97e+00
Halley		5	3.55e-15	2.48e-06	2.86e+00
M2		5	0.00e+00	1.27e-04	3.21e+00

Homeier	$x_0 = 8.7$	6	0.00e+00	6.66e-07	3.05e+00
Kou		7	0.00e+00	6.60e-11	2.97e+00
Halley		6	0.00e+00	2.97e-07	2.91e+00
M2		7	0.00e+00	8.44e-15	3.94e+00
Homeier	$x_0 = 12$	8	-3.55e-15	1.01e-08	3.05e+00
Kou		9	0.00e+00	8.77e-12	2.98e+00
Halley		8	0.00e+00	2.21e-10	2.97e+00
M2		9	0.00e+00	3.95e-14	3.93e+00

With the first two approximations, all methods make the same number of iterations, except the Kou method. By moving away from the root, the Kou and Homeier methods are distinguished by one iteration less than the other two.

Function 2: The graph of this feature shows that it has three real roots. Due to the constraints imposed on the initial approximation, we look for the root $x_n = 4.306584e+00$, since on the right along the Ox axis it is not limited by the first and the second derivative and the function and itssecond derivative have the same sign.

$f_2(x) = e^x - 4x^2$ in interval (3.5, 14), in which we look for root.

Table 2.

$f_2(x) = e^x - 4x^2$ $x_n = 4.306584e+00$				10e-15	
		it	$\|f(x_n)\|$	δ	ρ
Homeier	$x_0 = 11.9$	9	1.42e-14	0.00e+00	Inf
Kou		9	1.42e-14	0.00e+00	Inf
Halley		8	1.42e-14	5.33e-15	3.00e+00
M2		9	1.42e-14	0.00e+00	Inf
Homeier	$x_0 = 13.9$	10	1.42e-14	0.00e+00	Inf
Kou		10	-1.42e-14	3.55e-15	2.99e+00
Halley		9	1.42e-14	5.33e-15	3.01e+00
M2		10	1.42e-14	0.00e+00	Inf

With this feature, Halley's method makes an iteration less than the others. It requires calculating a second derivative, but at approximations that are far from the root it also faster than the others.

Function 3. We consider the nonlinear equation $f_3(x) = x^2 - e^{-x} - 3x + 2$ in interval (-7, -2), in which we look for root $x_n = -2.99223e+00$.

The function has three real roots, one of which is $x_n = -2.99223e + 00$. We will examine the function within the range (-7, -2), where the function and the second derivative of function pass under the axis Ox, i.e they have a negative sign that satisfies the condition $f(x_0).f''(x_0) > 0$

Table 3.

$f_3(x) = x^2 - e^{-x} - 3x + 2$	$x_n = -2.99223e + 00$	10e-15			
		it	$\lvert f(x_n)\rvert$	δ	ρ
Homeier	$x_0 = -6.5$	5	1.78e-15	4.61e-06	3.03e+00
Kou		6	1.78e-15	1.79e-09	2.95e+00
Halley		5	1.78e-15	6.86e-06	2.84e+00
M2		6	1.78e-15	1.02e-14	3.94e+00
Homeier	$x_0 = -6.6$	5	1.78e-15	1.02e-05	3.02e+00
Kou		6	1.78e-15	1.79e-09	2.95e+00
Halley		5	8.88e-15	1.38e-05	2.82e+00
M2		6	1.78e-15	1.57e-13	3.92e+00

Function 4. We consider the nonlinear equation $f_4(x) = x.\exp(x) - 1$, which has one real root $x_n = 567.143290e - 003$. To the left of the root, the first and the second derivative of the function are reset, so we choose to examine the function in the range$(-1, +\infty)$. Table 4 presents the results in a function study f_5, with three initial approximations selected to the right of the root to meet the condition $f(x_0).f''(x_0) > 0$.

Table 4.

$f_4(x) = x.\exp(x) - 1$	$x_n = 567.143290e - 003$	10e-15			
		it	$\lvert f(x_n)\rvert$	δ	ρ
Homeier	$x_0 = 2$	4	0.00e+00	4.24e-07	3.03e+00
Kou		5	0.00e+00	1.63e-14	2.99e+00
Halley		4	0.00e+00	5.24e-07	2.94e+00
M2		4	0.00e+00	2.36e-06	3.56e+00
Homeier	$x_0 = 5$	6	0.00e+00	1.28e-06	3.03e+00
Kou		7	2.22e-16	5.73e-11	2.98e+00
Halley		6	0.00e+00	2.50e-07	2.95e+00
M2		7	0.00e+00	2.99e-14	3.94e+00

With approximation $x_0 = 2$ again the Kou method has more iterations. The other methods make 4 iterations. With the next approximation x_0 =5, of the methods in which no second derivative is calculated, the Homeier method makes an iteration less than the Kou method, and from the methods requiring calculation of a second derivative, the Halley method, again has a smaller number of iterations (Homeier, 2004).

Function 5. We consider the nonlinear equation $f_5(x) = (x+2)e^x - 1$ in nterval (-1, 6), in which the root $x_n = \ -4.428544e - 01$ is sought.

The function is reset to the point $x_n = \ -4.428544e - 01$. The graphs of the first and the second derivatives go to the left of the root near the Ox. We examine the function in an interval (-1, 6) in which the two derivatives are not reset and for any point in this interval $f(x_0).f''(x_0) > 0$ is in effect.

Table 5.

$f_5(x) = (x+2)e^x - 1$	$x_n = \ -4.428544e - 01$				10e-15
		it	$\lvert f(x_n)\rvert$	δ	ρ
Homeier	$x_0 = 3.1$	6	0.00e+00	1.05e-14	3.01e+00
Kou		6	0.00e+00	1.12e-09	2.98e+00
Halley		5	0.00e+00	1.86e-06	2.95e+00
M2		6	0.00e+00	4.25e-12	3.91e+00
Homeier	$x_0 = 4.9$	7	0.00e+00	6.54e-13	3.01e+00
Kou		7	0.00e+00	1.05e-07	2.95e+00
Halley		6	0.00e+00	9.58e-08	2.97e+00
M2		7	0.00e+00	1.93e-08	3.77e+00
Homeier	$x_0 = 6$	7	0.00e+00	2.29e-06	3.03e+00
Kou		8	0.00e+00	3.04e-10	2.98e+00
Halley		7	0.00e+00	5.16e-09	2.98e+00
M2		8	0.00e+00	1.95e-11	3.89e+00

With the first two approximations, Halley's method makes the least number of iterations. The Homeier’s method is better than all the methods that do not require calculation of the second derivative. When comparing the other two methods that require second derivate calculation, Halley's method is better.

CONCLUSIONS.

In the present article, we review and compare iterative methods with order of convergence 3 and 4. Two of them require calculation of a second derivative. We experiment with 5 functions to find a real root. For initial approximations, we chose points in a space that meets Newton's terms.

When comparing the two methods that do not require calculation of a second derivative, the experiments show that the Homeier method makes less iterations than the Kou method, and when comparing the other two methods, Halley's method is faster than the method proposed by Farooq in article (Farooq, 2014; Parandjani et al, 2014).

Conflict of interest.

The author confirms that this article content has no conflict of interest.

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